DOCUMENT RESUME

ED 215 876 SE 037 067

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TITLE The Use of Small Group Protocols in Analysis of Children's Arithmetical Problem Solving.
PUB DATE [82]
NOTE 25p.

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Educational Research; Elementary Education; *Elementary School Mathematics; Grade 5; Grade 6; *Grouping (Instructional Purposes); *Learning Theories; Mathematics Achievement; *Mathematics Instruction; *Problem Solving
IDENTIFIERS Heuristics; *Mathematics Education Research

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THE USE OF SMALL GROUP PROTOCOLS IN ANALYSIS OF CHILDREN'S ARITHMETICAL PROBLEM SOLVING

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SE037 067

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Abstract

Strengths of four-person protocols as contrasted with the clinical interview and two-person protocols are described in the following areas: identification of odd and idiosyncratic heuristics revealed through reflexive talk; generalizable results that emerge from prolonged discussion; opportunities to study learning in small groups. Study describes discourse of fifth and sixth grade children solving arithmetical word problems in small groups without teacher intervention.

The Use of Small Group Protocols in Analysis of
Children's Arithmetical Problem Solving

Researchers in both psychology and mathematics education have shown increasing interest in the mental processes that contribute to competent performance in mathematics (Glaser, 1977). Increased activity in this area may be traced to the influence of Piaget and developmental psychology, to the use of computers in simulating mental processes (Newell and Simon, 1963; Greeno, 1978) and, perhaps, to discouragement with the slow results obtained in construct studies and a concomitant search for more pedagogically useful methods (Trivett, 1981).

The method that is central to the study of mental processes, often called the "clinical method" or "clinical interview," has evolved in a variety of forms from the method of overt thinking (Claparede, 1946). Using overt thinking in its pure or unchanneled form, the interviewer presents a problem and asks the subject to think aloud while solving it. "Let me hear you think" is the opening plea of the researcher or teacher trying to understand a student's mental functioning. One obvious difficulty with the method in this form -- particularly for the researcher who is usually a stranger to the subject -- is that the subject may remain silent. Because subjects are often more willing to speak after they have obtained a solution, interviewers sometimes ask for an account of the process in retrospect. But this introduces contamination from remembering, re-inventing, and editing. Further, subjects may often be unaware of the mental processes they

are actually employing. While introspection is still used and is sometimes highly productive with young children, the difficulties and limitations of the method are well known (Bruner, Goodnow, and Austin, 1962).

A study of the clinical method as it has evolved in psychogenetic and gestalt psychologies (Gobar, 1968) shows a steady trend toward channeling the interview in order to obtain speech from the subject and to direct that speech toward the interests of the researcher. While it seems right to say that the typical Piagetian interview is flexible in both construction and presentation of tasks and contingent upon the subject's responses (Ginsburg, 1981), it also seems right to note that the interviewer's comments and questions may direct subjects to ideas they would not otherwise have considered and that the channeling itself may constrain the use of odd, irrelevant, and highly inventive heuristics that might be revealed in the pure mode. Further, the one-to-one relationship that Ginsburg properly cites as a "strength" of the clinical interview contains an opposing weakness. The interviewer is unavoidably an authority, and this authoritative presence may also constrain the free use of available heuristics.

In spite of the recognized difficulties with variations of the clinical method, there is a growing body of such studies (Easley, 1977), and increased attention is being focussed on problems of the validity, reliability, and generalizability of their results (Swanson et al, 1981). Ginsburg (1981) suggests that the clinical method is used for three main purposes: 1) discovery of cognitive activities, 2) identification of such processes, and 3) evaluation of levels of competence. In the following discussion, I suggest that four person protocols may also be used for the discovery of cognitive activities.

This method removes the authority and, thus, exhibits some power in revealing odd heuristic moves that might not appear in the one-to-one situation. Furthermore, the method may be used to investigate learning in small groups (Barnes and Todd, 1977) and shows considerable promise as a pedagogical tool. Finally, the method may reveal generalizable results, and I shall discuss one of these.

Purposes of the larger study

In the larger project, which this paper describes in part, it was our aim to study how children approach mathematical word problems when there is no teacher intervention. We were interested particularly in the social-procedural moves they would use and in the mathematical heuristics they would employ. The dual instruments we constructed were designed to capture both their socio-linguistic activity and their mathematical activity, the latter concentrating chiefly on heuristics as organized into stages by Polya (1945). There is some controversy concerning the usefulness of these general heuristics. Lesh (1981), for example, drawing on results from several studies (Elstein et al, 1978; Schoenfeld, 1979; Simon and Simon, 1978; Larkin, McDermott, Simon and Simon, 1980) notes that <u>good</u> problem solvers more often use problem-specific heuristics and that <u>poor</u> problem solvers seem to be more likely to use the "powerful" general heuristics. This contention is at least partially supported by other research on the nature of heuristics used by problem solvers (Kilpatrick, 1967; Lucas, 1972; Webb, 1975; Kantowski, 1977; Pereira-Mendoza, 1979). But this result may well be an artifact of schooling. Good problem solvers will, of course, categorize problems, and once a problem is "typed," it is quite natural to use the method associated through effective

learning with the type. In unfamiliar contexts, however, it is likely that all problem solvers must depend upon general heuristics. The children whom we studied had been exposed to almost no instruction in word problems, and we wanted to see which heuristics they would employ and with what effect.

In this paper, because of the methodological focus, I shall not describe either the instruments or the quantitative results in any detail. Instead, I shall concentrate on the methodological strengths of the four-person protocol.

Procedures

Twenty-four students were identified from the fifth and sixth grade student populations of one school. The students were formed into six groups which, with the single exception of one all-girl group, were balanced with two boys and two girls. Each of the selected students belonged to the middle third of the local fifth and sixth grade distributions on both math and reading grade level equivalents as measured by the California Test of Basic Skills. Since the school's median placed it at about the second percentile state-wide, our subjects were performing considerably below grade level. Teachers were interviewed informally to ascertain that English was the dominant language of each of the 24 students. A measure of sociability was also obtained in the teacher interviews and, although the measure was not used to select student participants, it was reviewed later when decisions were made regarding student placements into the small groups. The students were assigned so that between any two group members there was a spread of no more than 1.5 years in the grade level equivalents for either math or reading. This was done to avoid the well known status effects of good

readers or good math students dominating group discussion (Cohen, 1980).

The groups were assembled once a week for a total of four sessions to record discussion of the word problems. Students were called out of their classrooms and directed to a table in one of three locations -- the media center, the school cafeteria or the back of a primary reading lab. Each session was divided roughly into two segments: the problem solving segment and the debriefing segment. During the problem solving segment, students rotated the responsibility for both reading the problem aloud and recording the agreed-upon solution on a group answer sheet. During the debriefing segment which followed, students were instructed to compare their answers to the solution key provided by a facilitator. They were told to go back to any problems they had missed and to figure out how they could get the correct answer. These were the only directions students received. In all other ways, student dialogue remained undirected, and the facilitators intervened only occasionally to encourage students to verbalize what they were thinking. It was the major responsibility of one facilitator to make such comments and of the other to take observational notes. At the end of each session, student scratch papers were gathered and labeled for later reference and the students were escorted back to class.

The design of our debriefing sessions went through several revisions during pilot exercises. At first, we felt a responsibility to inform the children on the correct answers and, also, to explain how the answers could be obtained. We quickly realized, however, that such instruction would surely confound our findings. While we were not trying to prevent learning effects -- the children would obviously and ideally learn from each other -- we did want to control "teacher" effects. The only solution seemed to be

to use the same procedures for both problem solving and debriefing segments. Thus the children themselves made the decision on whether an answer was acceptable and whether, in the debriefing session, a given explanation was adequate for the recorded answer.

Design of the problems

Each group of children worked eight problems in each of their four sessions. The problems were taken from a computer-based curriculum, Problem Solving (Noddings, 1978). The problems in this curriculum are carefully organized by reading level and by level of structural difficulty (Searle, Lorton, and Suppes, 1974; Loftus and Suppes, 1972; Loftus, 1970). The problems presented to any group over the four sessions were variants on eight "generic forms," and the problems appeared in different orders at each session. A typical generic form generates many specific problems that differ in superficial features.

In Problem Solving, the problems appear at three levels of structural difficulty. Only the simplest level problems were used in this study, and vocabulary was carefully controlled in order to keep the nonmathematical skills and peripheral information required to a minimum. Further, problems were selected so that their computational requirements lagged slightly behind the children's measured achievement scores. No attempt was made, however, to make the problems more "natural" or relevant to real world situations, even though several researchers have pointed to the artificiality of typical textbook problems as a source of student difficulty (Nesher, 1980; Lesh, 1981). Rather, the problems were chosen from "strands" that represent

themes typically found in school texts and standard tests. The dialogue reported here is taken from groups working at the 4.0 - 4.5 grade level.

Reflexive talk and individual thinking

One advantage of three or four person protocols over two person protocols is that individuals may talk aloud to themselves during group sessions without seeming odd or rude. In ordinary two person conversations, our partners would desert us if we persisted in talking aloud to ourselves. In small group problem solving, however, there are two foci: other persons with whom we must communicate and the problems themselves. Often children engaged in problem solving will express themselves aloud, receive and ignore a response from another child, and go on talking to themselves. We have labeled this sort of talk, "reflexive talk."

In this discussion I shall concentrate on one problem which was, apparently, new to all of the children. In the selected sequences of reflexive talk, we see several patterns of individual thinking. The problem that baffled the children was this one:

> Here is a number: 8,742. Use these four digits
> to form the number with the greatest value.

Now, of course, the problem appeared with a variety of initial digits, and sometimes we asked for the number with the "least" value. But one group, Group 3, did see the problem in this specific form and, since there was no change required, the problem was especially baffling to them. Their debate on first encounter with this problem consisted of 221 "acts"; by the fourth session, at which point they understood exactly what was required, their

debate was completed in 23 acts, and the correct answer actually emerged just four acts after the problem was read correctly.

In session one, Doug -- who eventually breaks the problem -- shows intense signs of concentration throughout the conversation. Although the other children listen to him, he often seems not to hear them. I shall include the numbering of acts here to give an indication of how the discourse is spread out. Early on, the other children are drawn into a debate about the shape of the "greater than" sign, but Doug persists in talking about the central issue:

32: Donny : Oh, you go like that, huh?
33: Doug : Wait.
34: : Using these 4 digits.
35: Donny : I know it's something like that.
36. Doug : . . . to form the number with the greatest . . .
37. Betty : I know, something like the greater . . .
38. : Or less
39. : I don't get it.
40. Donny : I know it's something that goes like . . .
41. : uh, . . . like . . .
42. : um *Donny is drawing "> "*
43. Doug : 4 digits.

While Betty and Donny debate the form of the "greater than" sign, Doug doggedly pursues the problem. In acts 92-95 he says again, "4 digits. No, wait. Wait. 4 digits . . . " By act 124 they are all exasperated.

Betty volunteers the re-read heuristic -- one that is powerful and, also, typical for Betty:

125. Betty : OK, read it again.

126. : Read it again.

127. : Read it again.

Immediately after the re-reading, Doug comes up with something the others laugh at:

132: Doug : 9, 9, 9, 9.

133. Betty : 9?

134. : Where did you get 9 from?

laughter

135. : They no 9 there.

136. : 8 is greater than 7, 4, and 2.

137. : But you have to use the four num . . .

138. Doug : All of the 9's.

139. Betty : *laughing* All the 9's.

140. : There isn't any 9's.

141. Doug : I don't get it.

But while Doug expresses bafflement, he really has it. Apparently, he has asked himself what the largest four-digit number is. His answer -- the only evidence of his "question" -- is "9,999." Very shortly after his dismayed, "I don't get it," we hear:

152: Doug : Use these four digits to form the number into the greatest value.

153. : Oh . . .

154. : Wait . . .

155. : It <u>is</u> the greatest value, I think.

There follows an odd side-sequence in which the children debate whether 7 is greater than 8. This can only be explained in the context of an implicit reference to place value. Doug remains focussed:

162. Betty : 1, 2, 3, 4, 5, 6, 7, 8.
163. Donny : Oh, oh, oh.
164. : 8 is bigger.
165. : Ha?
166. Doug : It <u>is</u> the . . . the greatest.
167. Betty : OK.
168. : Do you agree?
169. : Agree?
170. Doug : No.

This inexplicable "no" from the person who offered the answer sends Betty and Donny into hushed conversation about Doug's withholding.

176. Doug : Use these four digits to form the number with the greatest value . . .
177. : That <u>is</u> the greatest value.
178. Betty : You . . . *laughter*
179. : Maybe we have to add a couple of these numbers together.
180. Doug : That <u>is</u> the greatest value.
181. : So what is it?

Here Doug seems to be asking what the problem makers could possibly have had in mind. Here <u>is</u> the answer. There is nothing to do. So what should we do? Doug then checks his thinking.

184. Doug : It can't be 7.

185. : It can't be 4.

186. : It can't be 2.

A little later he decides:

196. Doug : I agree.

197. : I agree.

198. : I agree!

The session closes with sighs of frustration and Donny's directions to Betty who writes the answer:

220. Donny : Like it is.

221. : Like it is.

Doug's engagement persists into the debriefing session. Usually the children spot the problems they got wrong immediately and concentrate their attention on those problems. This time, Doug has only one thing in mind. As soon as the answer sheets appear, Doug speaks:

Doug : Oh, no.

: Oh!

: I knew it!

: It was 8, 7, 4, 2.

: See?

: I knew it!

: Right there.

: 8, 7, 4, 2.

Betty : See?

: Well, it says leave it as it is.

Donny : We got it right?

Betty : It says, leave it as it is.

Donny : I got that wrong?

Betty : It says leave it as it is.

Doug : I knew it.

Betty : That's the way it is on the paper.

Doug : I knew it was right.

At this point, Betty offers an explanation for what is going on:

Betty : Because it's

: it's . . .

: There's no . . .

: 7's not greater than 8.

: and 4's not greater than 8 . . .

: and 2's not greater than 8.

: So, just leave it as it is.

Doug : 8

: 8742

: That's the greatest it can go.

In this first session, both Donny and Betty made attempts -- far more superficial than Doug's -- to break the problem quickly. Donny, for example, tried to interpret the problem:

27. Donny : That means circle the one with the highest value . . .

28. : I think.

And, Betty, as we recorded in her act 179, suggests:

179. Betty : Maybe we have to add a couple of these numbers together.

Such "classroom" heuristics are not unusual. The children often abandon mathematical thinking and, instead, ask what the teacher or curriculum maker might be seeking.

Another example of reflexive talk occurs in Group 2. In their initial debate, which occupied 166 acts, Harry had the right answer at act 22. But Jim was totally absorbed in trying to interpret the problem. The problem this time:

> Here is a number: 5,963. Use these 4 digits to form the number with the least value.

After initial expressions of bafflement and calls for re-reading, Jim says:

19.	Jim	:	Oh!
20.		:	I . . . I know the answer now.
21.		:	Wouldn't it be . . .
22.	Harry	:	3,569
23.	Jim	:	Wouldn't it be this one right here . . .
24.	Harry	:	No.
25.	Jim	:	5 9 6 0
26.	Harry	:	Nope.
27.	Jim	:	5 9 . . .
28.		:	Yeah.
29.		:	5 9
30.		:	. . . 6
31.		:	Just take out the last one for the value.

Here we see what may be a typical heuristic. Jim is trying to make sense of the problem by relating key words such as "digits" and "value" to

previously encountered problems. Apparently he has seized on a "rounding" procedure as the key to this problem: just take out the last one for the value and get 5960.

Another youngster, Anna, seems to realize that the problem has something to do with place value, but she gets bogged down in the labels for columns:

60. Anna : I got . . .
61. : Wait . . .
62. : That's 10's, 100's
63. : Wait.
64. : 1's, 10's, 100's, 1000's, . . .
65. : What's after thousands?
66. : Millions?
67. : Ten thousand
68. : Ten thousand.

By act 120, Jim -- still ignoring Harry who has given up trying to get them to listen ("Let 'em struggle!") -- resorts to looking for the smallest digit:

120. Jim : Look.
121. This one would be it . . .
122. : Right?
123. : It says "the least value."
124. : 3 is the less.

A few steps later, by building on what Jim actually says, Harry gets him to listen:

136. Jim : It would be this . . .

137. : The least value . . .

138. : All right . . .

139. : 3

140. : The lesser of all these . . .

141. : Right?

142. Harry : . . . then comes 5.

143. Jim : And then comes 5, . . .

144. Harry : Then comes . . .

145. Jim : Then comes 6.

146. : Then 9.

147. Harry : That's what I had.

148. : That's what I explained to ya.

This sequence illustrates vividly how effective it is instructionally to build on the student's own thinking. Nothing Harry said earlier could break through Jim's intense personal engagement.

In all of our groups, we heard attempts to use "quick-trick" heuristics but, possibly because no authority was present to assent or dissent to these attempts, they were usually abandoned. Intense engagement and persistent reflexive talk eventually captured the group and steered them to consensus.

Generalizable effects

When children are discussing problems on their own and no interviewer channels their thinking, we might suppose that idiosyncratic thinking would be revealed (as, indeed, it was in their reflexive talk) but that few

generalizable results would appear. We shall certainly be able to suggest something about the number and variety of heuristics children employ, but here I shall discuss only one result that occurred in every group we studied.

The children we studied, even though many of them were sixth graders, had very little understanding of fractions. They had heard the word "fraction" but could not think in terms of thirds, sevenths, ninths, and the like. The problem that stymied them was this:

> James ate 1/3 of a pizza. How much did he have left?

The fraction varied, and "pizza" was sometimes replaced by "pie" or "cake." In <u>every</u> group, sooner or later, someone asked about the absolute size of the pizza, pie, or cake. (We speculated that this might have been caused by the wording of the question, i.e., "how much" rather than "what part," but a check of difficulties in later sessions showed that this was probably not the case.)

In Group 2, after Anna has guessed "4/3" and Jim has tried "a half," Jim tries to explain:

19.	Jim	:	Put this . . .
20.		:	There, there's three.
21.		:	1, 2, 3, . . .
22.		:	Wait, no.
23.		:	What am I doing?
24.	Harry	:	How big a pizza anyway?

In this first session, Group 2 finished up with "2 halves" as an answer.

Group 5 had to cope with someone's eating 1/5 of a pizza. These children almost gave up on the problem. After 55 acts, they decided to come back to

the fraction problem after completing the others. In the second attempt, we hear again the heuristic we have since labeled, "refer to absolute size":

13. Tommy : It's one whole pizza
14. : And she takes away 1/5 of it.
15. Sammy : And it asks ya how much would be left?
16. Tommy : How much would be left.
(Much debate.)
36. Tommy : I would have done it like this.
37. : Think.
38. : But I don't know what . . . how big the pizza was.
39. Tommy : It doesn't say how big the pizza was.
41. : It was medium, small, or large?
(Much debate.)
62. Sammy : If she was by herself it would be small.
64. Joan : It doesn't mention anybody else.
65. Sammy : Well, let's just put it like . . . by herself.

The group debated a bit more and, in act 80, decided to leave the answer blank. This is the only time a group made that decision. The children were remarkably determined to get answers and, in Group 3, when the suggestion was made to leave an answer blank, Betty responded, "We can't do that. We <u>have</u> to get it!"

But Group 3 had trouble, also, when "Maria ate 1/7 of a cake."

10. Doug : 6.
11. Betty : Maria ate 1/7 of a cake . . .
12. Doug : <u>6</u>.

13. Betty : Doesn't matter.

14. : How big is the cake?

15. Doug : Six pieces left.

16. Betty : You don't know how big the cake is.

Throughout the discussion (55 acts), Doug reasserts, "6," and Betty (Acts 24, 37) asks, "How big is the cake?" The group finished with Doug's answer, "6." It is interesting to note that the "refer to absolute size" heuristic appeared in every group, and it was never challenged as irrelevant even though some children, e.g., Doug, would not have suggested it themselves.

This result suggests that well know difficulties with fractions are widespread and very basic. In related research, Larson (1980) found that students had more difficulty locating "1/5" on a two-unit segment than on a one-unit segment. The discourse that we recorded suggests that the same "absolute size" heuristic may be operating in both task situations.

Learning in small groups

The four person protocol is useful in research on mental processes. The authority is removed, and reflexive talk reveals odd and interesting heuristics. Further, discussion is sufficiently prolonged so that certain heuristics are suggested over and over with remarkably similar reception across groups. But beyond its usefulness for research, the small group working without teacher intervention seems to be a powerful learning tool. All of our groups finished their fourth and final session with all answers correct.

The children obviously figured things out and learned from each other. Although there was no <u>teacher</u> intervention, it is important to note that there was substantial <u>instructional</u> intervention. The problems themselves were

carefully structured and maintained in disguise over the four sessions. The children did receive the answers, and they discussed them fully in their debriefing sessions. But the absence of a teacher/evaluator threw the children on their own resources. The resulting rejection of nonmathematical heuristics ("Maybe we should add a couple of these numbers"; watching facial expressions while asking, "Add?") surprised and pleased us. Further, the genuine engagement and pleasure that the children displayed suggest that word problems need not be either terrifying or boring.

An obvious and important question that we hope to address in the next stage of our research is whether small group methods of this sort result in substantial improvement in individual problem solving achievement. Existing research on this question is not promising (Johnson et al, 1981), but most studies have reported very limited treatment periods, and some treatments have been narrowly and unnecessarily constrained.

Finally, one important weakness in small group methods should be mentioned. It was often the case in our groups that one child withdrew and spoke only when addressed directly. Clearly, we know very little about the thinking processes of these quiet children. It may be that even these children learned quite a lot. Bayer (1982), in a different setting -- working with adult group members on language policies -- reports that a quiet member revealed an impressive level of learning in her final written report. This possibility provides further reason for looking next at individual changes in achievement.

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